

SOBOLEV SPACES IN MATHEMATICS I

SOBOLEV TYPE INEQUALITIES

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SOBOLEV SPACES IN MATHEMATICS I

Sobolev Type Inequalities

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*To the memory of
Sergey L'vovich Sobolev
on the occasion of his centenary*

Main Topics

Sobolev's discoveries of the 1930's have a strong influence on development of the theory of partial differential equations, analysis, mathematical physics, differential geometry, and other fields of mathematics. The three-volume collection *Sobolev Spaces in Mathematics* presents the latest results in the theory of Sobolev spaces and applications from leading experts in these areas.

I. Sobolev Type Inequalities

In 1938, exactly 70 years ago, the original Sobolev inequality (an embedding theorem) was published in the celebrated paper by S.L. Sobolev "On a theorem of functional analysis." By now, the Sobolev inequality and its numerous versions continue to attract attention of researchers because of the central role played by such inequalities in the theory of partial differential equations, mathematical physics, and many various areas of analysis and differential geometry. The volume presents the recent study of different Sobolev type inequalities, in particular, inequalities on manifolds, Carnot–Carathéodory spaces, and metric measure spaces, trace inequalities, inequalities with weights, the sharpness of constants in inequalities, embedding theorems in domains with irregular boundaries, the behavior of maximal functions in Sobolev spaces, etc. Some unfamiliar settings of Sobolev type inequalities (for example, on graphs) are also discussed. The volume opens with the survey article "My Love Affair with the Sobolev Inequality" by David R. Adams.

II. Applications in Analysis and Partial Differential Equations

Sobolev spaces become the established language of the theory of partial differential equations and analysis. Among a huge variety of problems where Sobolev spaces are used, the following important topics are in the focus of this volume: boundary value problems in domains with singularities, higher order partial differential equations, nonlinear evolution equations, local polynomial approximations, regularity for the Poisson equation in cones, harmonic functions, inequalities in Sobolev–Lorentz spaces, properties of function spaces in cellular domains, the spectrum of a Schrödinger operator with negative potential, the spectrum of boundary value problems in domains with cylindrical and quasicylindrical outlets to infinity, criteria for the complete integrability of systems of differential equations with applications to differential geometry, some aspects of differential forms on Riemannian manifolds related to the Sobolev inequality, a Brownian motion on a Cartan–Hadamard manifold, etc. Two short biographical articles with unique archive photos of S.L. Sobolev are also included.

III. Applications in Mathematical Physics

The mathematical works of S.L. Sobolev were strongly motivated by particular problems coming from applications. The approach and ideas of his famous book “Applications of Functional Analysis in Mathematical Physics” of 1950 turned out to be very influential and are widely used in the study of various problems of mathematical physics. The topics of this volume concern mathematical problems, mainly from control theory and inverse problems, describing various processes in physics and mechanics, in particular, the stochastic Ginzburg–Landau model with white noise simulating the phenomenon of superconductivity in materials under low temperatures, spectral asymptotics for the magnetic Schrödinger operator, the theory of boundary controllability for models of Kirchhoff plate and the Euler–Bernoulli plate with various physically meaningful boundary controls, asymptotics for boundary value problems in perforated domains and bodies with different type defects, the Finsler metric in connection with the study of wave propagation, the electric impedance tomography problem, the dynamical Lamé system with residual stress, etc.

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Sobolev Type Inequalities

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